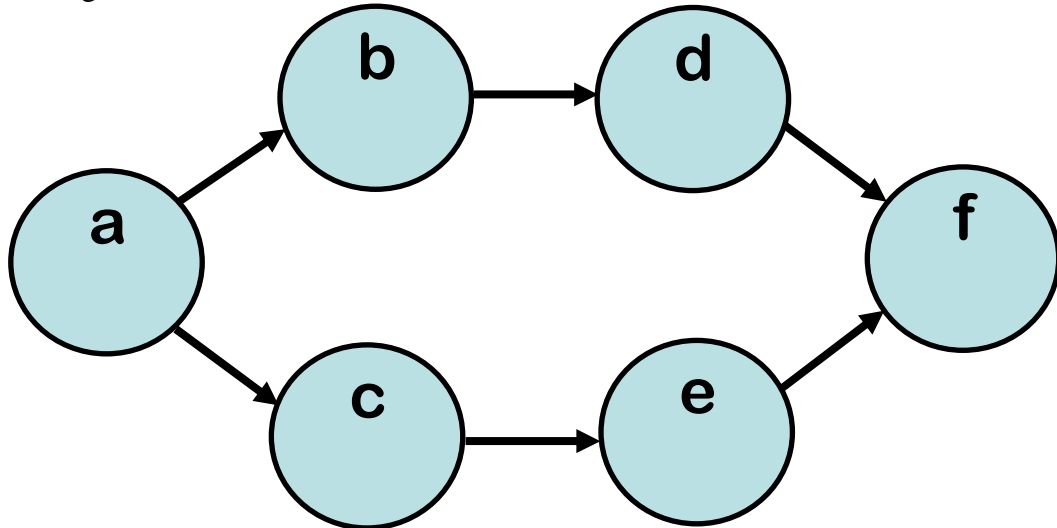Problem Solving

„Crashing"

Problem 1
A firm has a small project with 6 'elemental tasks', labelled with small fonts from 'a' to 'f'.
Logical (precedential) connections among them are illustrated by the following simplified
network diagram:

Additional data are collected in the table below about the activities:

Label	Original duration (days)	Minimum duration (days)	Cost slope (dollars/day)	Float (days)
a	2	1	100	
b	4	3	50	
c	2	1	50	
d	4	3	150	
e	3	2	20	
f	3	3	–	

1.a. Identify the activity paths and the Total Project Time (TPT)!Calculate the float times of the activities with the help of the network diagram!
1.b. Calculate the minimum TPT available through crashing and its total cost with the help of the following table!

	Steps of crashing						
	1	*2*	*3*	*4*	*5*	*6*	*7*
Crashed activity							
Crashing cost							
Σ crashing cost							
a-b-d-f							
a-c-e-f							
TPT							

SOLUTION

1.a.

First we have to identify the activity paths. We can find two of them: *a-b-d-f* and *a-c-e-f*. Then calculate the duration of the paths via summing up the duration of the individual activities on the given path. *a-b-d-f* = 13 days, *a-c-e-f* = 10 days. The longest one is called critical path and the activities on the critical path (or paths) are the critical activities. The difference between the paths' durations is the float (and it is 3 days). Float is connected to the non-critical activities. In this exercise it is indifferent on which non-critical activity will we place the float, thus we have to record 3 days at both *c* and *e*. Critical activities has no float by definition. The duration of the critical path is the TPT (it is 13 days).

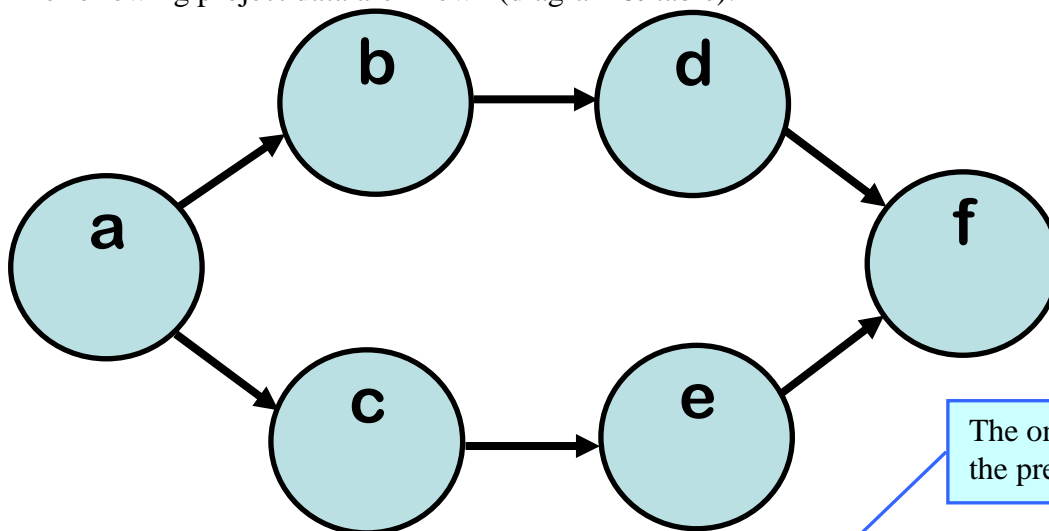| Label	Original duration (days)	Minimum duration (days)	Cost slope (dollars/day)	Float (days)
a	2	1	100	0
b	4	3	50	0
c	2	1	50	3
d	4	3	150	0
e	3	2	20	3
f	3	3	–	0

1.b.

How to 'crash' activities:

1. We crash critical activities only, because the aim is to decrease TPT.
2. We do crashing incrementally: only one day per step.
3. Only those activities are 'crashable' that has a minimum duration lower than the actual duration. After being crashed, the actual duration is lowered, and when it reaches the minimum, the activity is not available for further crashing.
4. Always crash the cheapest available critical activity.
5. If there are multiple critical paths we should crash all of them simultaneously. With crashing we are reducing duration of the critical paths, thus it is possible that the number of critical paths will increase as the original critical ones reach the duration of the longest non-criticals.
6. The process stops when crashing is not possible on any of the actual critical paths.

	Steps of crashing						
	1	*2*	*3*	*4*	*5*	*6*	*7*
Crashed activity	b	a	d				
Crashing cost	50	100	150				
Σ crashing cost	50	150	300				
a-b-d-f	12	11	10				
a-c-e-f	10	9	9				
TPT	12	11	10				

Problem 2

The following project data are known (diagram & table):

The only difference from the previous exercise.

Label	Original duration (days)	Minimum duration (days)	Cost slope (dollar/day)	Float (days)
a	2	1	100	
b	4	3	50	
c	2	1	50	
d	4	1	150	
e	3	2	20	
f	3	3	–	

2.a. Identify the activity paths and the Total Project Time (TPT)!Calculate the float times of the activities with the help of the network diagram!
2.b. Calculate the minimum TPT available through crashing and its total cost with the help of the following table!

	Steps of crashing						
	1	*2*	*3*	*4*	*5*	*6*	*7*
Crashed activity							
Crashing cost							
Σ crashing cost							
a-b-d-f							
a-c-e-f							
TPT							

<u>SOLUTION</u>

2.a. Same as the previous exercise.

2.b. The first 3 steps are the same, but in this exercise we can continue crashing activity 'd' because its minimum duration is lower.

	Steps of crashing						
	1	*2*	*3*	*4*	*5*	*6*	*7*
Crashed activity	b	a	d	d			
Crashing cost	50	100	150	150			
Σ crashing cost	50	150	300	450			
a-b-d-f	12	11	10	**9**			
a-c-e-f	10	9	9	**9**			
TPT	12	11	10	9			

In the 4th step the **durations of the two paths became the same**, which means that from this point we have two critical paths, thus all the activities are critical from now. Two critical paths can be crashed simultaneously if we crash an activity that is on both (like 'a' and 'f' in this problem) or one activity on each. The first solution is not feasible here because both 'a' and 'f' are at their minimum duration. But it is possible to crash 'd' one day further AND crash 'c' or 'e' simultaneously. Since the cost slope of 'e' is smaller, we have to choose the 'd+e' combination.

	Steps of crashing						
	1	*2*	*3*	*4*	*5*	*6*	*7*
Crashed activity	b	a	d	d	d + e		
Crashing cost	50	100	150	150	150+20 =170		
Σ crashing cost	50	150	300	450	620		
a-b-d-f	12	11	10	9	8		
a-c-e-f	10	9	9	9	8		
TPT	12	11	10	9	8		

The crashing process stops here, because the *a-b-d-f* path reached its minimum duration. *TPT = 8, total crashing cost = 620* dollars.

	Steps of crashing						
	1	*2*	*3*	*4*	*5*	*6*	*7*
Crashed activity	b	a	d	d	d + e		
Crashing cost	50	100	150	150	150+20 =170		
Σ crashing cost	50	150	300	450	620		
a-b-d-f	12	11	10	9	8		
a-c-e-f	10	9	9	9	8		
TPT	12	11	10	9	8		