

Product Design practicing exercises – paired comparison

Exercise 1. Determine the consistency coefficient (K) of the following decision makers.

a) Person 1

	I1	I2	I3	I4	I5	I6	I7	I8
I1		1	1	1	0	0	1	1
I2	0		0	1	0	0	0	1
I3	0	1		1	0	0	1	1
I4	0	0	0		0	0	1	1
I5	1	1	1	1		1	1	1
I6	1	1	1	1	0		1	1
I7	0	1	0	0	0	0		1
I8	0	0	0	0	0	0	0	

b) Person 2

	I1	I2	I3	I4	I5	I6	I7	I8
I1		0	0	0	0	0	0	0
I2	1		1	0	1	0	0	0
I3	1	0		1	1	0	0	0
I4	1	1	0		0	0	0	0
I5	1	0	0	1		0	0	0
I6	1	1	1	1	1		1	1
I7	1	1	1	1	1	0		0
I8	1	1	1	1	1	0	1	

c) Person 3

	I1	I2	I3	I4	I5	I6	I7
I1		1	1	1	0	0	1
I2	0		0	1	0	0	0
I3	0	1		1	0	0	1
I4	0	0	0		0	0	1
I5	1	1	1	1		1	1
I6	1	1	1	1	0		1
I7	0	1	0	0	0	0	

Exercise 2

	I1	I2	I3	I4	I5	I6	I7	I8
I1		7	5	7	2	3	4	4
I2	2		2	7	2	1	2	2
I3	4	7		9	2	3	4	2
I4	2	2	0		0	1	4	2
I5	7	7	7	9		5	5	3
I6	6	8	6	8	4		8	6
I7	5	7	5	5	4	1		2
I8	5	7	7	7	6	3	7	

- Determine the weights of the attributes based on the rows' distribution if 9 people give answer to the questionnaire, and all of them were consistent.
- Determine the preference fraction and the weights (on the scale between 1-10) of each item from the totalized preference matrix above.
- Determine the Kendall coefficient of concordance from the same totalized preference matrix, if 9 people give answer to the questionnaire, and all of them were consistent.

Exercise 3

14 customers were asked about a product's quality features. Then based on 9 consistent decision makers' answers, we created the summarized preference matrix (see below).

- Determine the weights of the attributes based on the rows' distribution.
- Determine the importance of each dimension (item) on a 1-10 scale (with the help of linear transformation).
- Determine the agreement of answerers. Calculate the Kendall's coefficient of concordance!

	I1	I2	I3	I4	I5	I6	I7	I8
I1		5	2	5	5	2	6	1
I2	4		1	9	1	3	2	0
I3	7	8		8	8	2	6	0
I4	4	0	1		1	1	6	0
I5	4	8	1	8		0	5	1
I6	7	6	7	8	9		8	3
I7	3	7	3	3	4	1		0
I8	8	9	9	9	8	6	9	

Exercise 4

14 customers were asked about a product's quality features. Then based on 9 consistent decision makers' answers, we created the summarized preference matrix (see below).

- Determine the weights of the attributes based on the rows' distribution.
- Determine the importance of each dimension (item) on a 1-10 scale (with the help of linear transformation).
- Determine the agreement of answerers. Calculate the Kendall's coefficient of concordance!

	I1	I2	I3	I4	I5	I6	I7	I8
I1		5	4	5	5	3	6	3
I2	4		3	9	3	5	3	1
I3	5	6		6	7	2	4	0
I4	4	0	3		1	1	5	0
I5	4	6	2	8		0	3	1
I6	6	4	7	8	9		8	2
I7	3	6	5	4	6	1		1
I8	6	8	9	9	8	7	8	

SOLUTION

Exercise 1

a)

Step 1. Calculate the sum of each row (a) and its square (a^2) – see in the table below, and summarize a^2 . $\rightarrow a^2=138$

	I1	I2	I3	I4	I5	I6	I7	I8	a	a ²
I1		1	1	1	0	0	1	1	5	25
I2	0		0	1	0	0	0	1	2	4
I3	0	1		1	0	0	1	1	4	16
I4	0	0	0		0	0	1	1	2	4
I5	1	1	1	1		1	1	1	7	49
I6	1	1	1	1	0		1	1	6	36
I7	0	1	0	0	0	0		1	2	4
I8	0	0	0	0	0	0	0		0	0

Step 2. Since n is an even number, we use $d_{\max} = \frac{n^3 - 4n}{24}$ to calculate the maximum number of trial decisions (where n is the number of items, n=8).

$$d_{\max} = \frac{(6^3 - 4 \cdot 6)}{24} = 20$$

Step 3. Calculate inconsistent trial decisions (d) from the following equation:

$$d = \frac{n(n-1)(2n-1)}{12} - \frac{\Sigma a^2}{2} = \frac{8 \cdot 7 \cdot 15}{12} - \frac{138}{2} = 70 - 69 = 1$$

Step 4. $K = \left(1 - \frac{d}{d_{\max}}\right) * 100\% \quad K = \left(1 - \frac{1}{20}\right) = 0,95 \rightarrow \boxed{95\%}$

b)

Step 1. $a^2=136$

	I1	I2	I3	I4	I5	I6	I7	I8	a	a ²
I1		0	0	0	0	0	0	0	0	0
I2	1		1	0	1	0	0	0	3	9
I3	1	0		1	1	0	0	0	3	9
I4	1	1	0		0	0	0	0	2	4
I5	1	0	0	1		0	0	0	2	4
I6	1	1	1	1	1		1	1	7	49
I7	1	1	1	1	1	0		0	5	25
I8	1	1	1	1	1	0	1		6	36

Step 2. $d_{\max}=20$

Step 3. $d = \frac{8 \cdot 7 \cdot 15}{12} - \frac{136}{2} = 70 - 68 = 2$

Step 4. $K = \left(1 - \frac{2}{20}\right) = 0,90 \rightarrow \boxed{90\%}$

c)

Step 1. $a^2=89$

	I1	I2	I3	I4	I5	I6	I7	a	a ²
I1		1	1	1	0	0	1	4	16
I2	0		0	1	0	0	0	1	1
I3	0	1		1	0	0	1	3	9
I4	0	0	0		0	0	1	1	1
I5	1	1	1	1		1	1	6	36
I6	1	1	1	1	0		1	5	25
I7	0	1	0	0	0	0		1	1

Step 2. Since n is an odd number, we should use $d_{\max} = \frac{(n^3 - n)}{24}$

$$d_{\max} = \frac{(7^3 - 7)}{24} = 14$$

Step 3. $d = \frac{7 \cdot 6 \cdot 13}{12} - \frac{89}{2} = 45,5 - 44,5 = 1$

Step 4. $K = \left(1 - \frac{1}{14}\right) = 0,92857 \rightarrow \boxed{92,86\%}$

Exercise 2

a)

We have to determine the sum of rows (a). After that calculate $\sum a$, and determine the percentage of each row's sum.

$\sum a=241$, thus

1. row: $\frac{32}{241} = 13,3\%$

2. row: $\frac{18}{241} = 7,5\%$ etc.

	I1	I2	I3	I4	I5	I6	I7	I8	a	%
I1		7	5	7	2	3	4	4	32	13,3%
I2	2		2	7	2	1	2	2	18	7,5%
I3	4	7		9	2	3	4	2	31	12,9%
I4	2	2	0		0	1	4	2	11	4,6%
I5	7	7	7	9		5	5	3	40	16,6%
I6	6	8	6	8	4		8	6	40	16,6%
I7	5	7	5	5	4	1		2	27	11,2%
I8	5	7	7	7	6	3	7		42	17,4%

Remember, that the sum of these percentages is always 100%.

b)

Step 1. Calculate preference fraction (P_a) from the next equation $P_a = \frac{a + \frac{m}{2}}{m \cdot n}$ where

a is the sum of the rows,
 n the number of items ($n=8$) and

m is the number of consistent decision makers ($m=9$).

	I1	I2	I3	I4	I5	I6	I7	I8	a	a+m/2	Pa
I1		7	5	7	2	3	4	4	32	36,50	0,5069
I2	2		2	7	2	1	2	2	18	22,50	0,3125
I3	4	7		9	2	3	4	2	31	35,50	0,4931
I4	2	2	0		0	1	4	2	11	15,50	0,2153
I5	7	7	7	9		5	5	3	40	44,50	0,6181
I6	6	8	6	8	4		8	6	40	44,50	0,6181
I7	5	7	5	5	4	1		2	27	31,50	0,4375
I8	5	7	7	7	6	3	7		42	46,50	0,6458

Step 2. Determine the relating z scores of Pa values in each row. Use the table of normal distribution.

	I1	I2	I3	I4	I5	I6	I7	I8	Pa	z
I1		7	5	7	2	3	4	4	0,5069	0,02
I2	2		2	7	2	1	2	2	0,3125	-0,49
I3	4	7		9	2	3	4	2	0,4931	-0,02
I4	2	2	0		0	1	4	2	0,2153	-0,79
I5	7	7	7	9		5	5	3	0,6181	0,3
I6	6	8	6	8	4		8	6	0,6181	0,3
I7	5	7	5	5	4	1		2	0,4375	-0,16
I8	5	7	7	7	6	3	7		0,6458	0,37

Step 3. With a $\frac{z_i - z_{\min}}{z_{\max} - z_{\min}} \cdot 100\%$ linear transformation we can determine how far it is from the minimal value of z 's

Where z_i is the z value of the row i

z_{\min} is the minimum of z 's ($z_{\min} = -0,79$)

z_{\max} is the maximum of z 's ($z_{\max} = 0,37$)

$z_{\max} - z_{\min} = 0,79 - (-0,37) = 1,16$

$$1. \text{ row: } \frac{0,02 - (-0,79)}{1,16} = 69,83\%$$

$$2. \text{ row: } \frac{-0,49 - (-0,79)}{1,16} = 25,86\%$$

$$3. \text{ row: } \frac{-0,02 - (-0,79)}{1,16} = 66,38\%$$

$$4. \text{ row: } \frac{-0,79 - (-0,79)}{1,16} = 0\%$$

$$5. \text{ row: } \frac{0,3 - (-0,79)}{1,16} = 93,97\%$$

$$6. \text{ row: } \frac{0,3 - (-0,79)}{1,16} = 93,97\%$$

$$7. \text{ row: } \frac{-0,16 - (-0,79)}{1,16} = 54,31\%$$

$$8. \text{ row: } \frac{0,36 - (-0,79)}{1,16} = 100\%$$

Step 3. Determine weights on the basis of the followings:

0-10,99 % → 1	50-59,99 % → 6
10-19,99 % → 2	60-69,99 % → 7
20-29,99 % → 3	70-79,99 % → 8
30-39,99 % → 4	80-89,99 % → 9
40-49,99 % → 5	90-100 → 10

	I1	I2	I3	I4	I5	I6	I7	I8	%	weights
I1		7	5	7	2	3	4	4	69,38	7
I2	2		2	7	2	1	2	2	25,86	3
I3	4	7		9	2	3	4	2	66,38	7
I4	2	2	0		0	1	4	2	0	1
I5	7	7	7	9		5	5	3	93,97	10
I6	6	8	6	8	4		8	6	93,97	10
I7	5	7	5	5	4	1		2	54,31	6
I8	5	7	7	7	6	3	7		100	10

c)

$$\text{Step 1. Calculaet } \Delta_{\max} = \frac{m^2 \cdot (n^3 - n)}{12} = \frac{9^2 - (8^3 - 8)}{12} = \frac{81 \cdot 504}{12} = 3402$$

Step 2. Summarize each column (R_j) and calculate the mean (\bar{R}_j)

$$\bar{R}_j = \frac{29 + 38 + 25 + 38 + 16 + 13 + 28 + 15}{8} = 25,25$$

Then calculate $(R_j - \bar{R}_j)^2$ for each column

- col: $(29 - 25,25)^2 = 14,06$
- col: $(38 - 25,25)^2 = 162,56$
- col: $(25 - 25,25)^2 = 0,06$
- col: $(38 - 25,25)^2 = 162,56$
- col: $(16 - 25,25)^2 = 85,56$
- col: $(13 - 25,25)^2 = 150,06$
- col: $(28 - 25,25)^2 = 7,56$
- col: $(15 - 25,25)^2 = 105,06$

	I1	I2	I3	I4	I5	I6	I7	I8
I1		7	5	7	2	3	4	4
I2	2		2	7	2	1	2	2
I3	4	7		9	2	3	4	2
I4	2	2	0		0	1	4	2
I5	7	7	7	9		5	5	3
I6	6	8	6	8	4		8	6
I7	5	7	5	5	4	1		2
I8	5	7	7	7	6	3	7	
Rj	29	38	25	38	16	13	28	15
(Rj-Rj_{mean})²	14,06	162,56	0,06	162,56	85,56	150,06	7,56	105,06

Step 3. Calculate

$$\Delta = \sum (R_j - \bar{R}_j)^2 = 14,06 + 162,56 + 0,006 + 162,56 + 85,56 + 150,06 + 7,56 + 105,06 = 687,48$$

Step 4. $K = \frac{\Delta}{\Delta_{\max}} = \frac{687,48}{3402} = 0,2021 \rightarrow \boxed{20,21\%}$

This means that there is a little concordance among decision makers. In this case other statistical analyses needed, for example cluster analyses to determine groups with same preferences.

Exercise 3.

a)

	a	%
I1	26	10,3%
I2	20	7,9%
I3	39	15,5%
I4	13	5,2%
I5	27	10,7%
I6	48	19,0%
I7	21	8,3%
I8	58	23,0%

b)

	a	Pa	z		Grade
I1	26	0,4236	-0,19	28,02%	3
I2	20	0,3403	-0,41	15,93%	2
I3	39	0,6042	0,26	52,75%	6
I4	13	0,2431	-0,7	0,00%	1
I5	27	0,4375	-0,16	29,67%	3
I6	48	0,7292	0,61	71,98%	8
I7	21	0,3542	-0,37	18,13%	2
I8	58	0,8681	1,12	100,00%	10

b)

	I1	I2	I3	I4	I5	I6	I7	I8
I1		5	2	5	5	2	6	1
I2	4		1	9	1	3	2	0
I3	7	8		8	8	2	6	0
I4	4	0	1		1	1	6	0
I5	4	8	1	8		0	5	1
I6	7	6	7	8	9		8	3
I7	3	7	3	3	4	1		0
I8	8	9	9	9	8	6	9	
Rj	37	43	24	50	36	15	42	5
$(R_j - R_{j_{\text{mean}}})^2$	30,25	132,25	56,25	342,25	20,25	272,25	110,25	702,25
Rj _{mean} = 31,5								
Δ _{max} = 3402								
Δ = 1666								
K = 48,97%								

Exercise 4.

a)

	a	%
I1	31	12,3%
I2	28	11,1%
I3	30	11,9%
I4	14	5,6%
I5	24	9,5%
I6	44	17,5%
I7	26	10,3%
I8	55	21,8%

b)

	a	a+m/2	Pa	z	%	Grade
I1	31	35,5	0,493	-0,0200	39,62%	4
I2	28	32,5	0,451	-0,1200	33,33%	4
I3	30	34,5	0,479	-0,0600	37,11%	4
I4	14	18,5	0,257	-0,6500	0,00%	1
I5	24	28,5	0,396	-0,2600	24,53%	3
I6	44	48,5	0,674	0,4500	69,18%	7
I7	26	30,5	0,424	-0,1900	28,93%	3
I8	55	59,5	0,826	0,9400	100,00%	10

c)

	I1	I2	I3	I4	I5	I6	I7	I8
I1		5	4	5	5	3	6	3
I2	4		3	9	3	5	3	1
I3	5	6		6	7	2	4	0
I4	4	0	3		1	1	5	0
I5	4	6	2	8		0	3	1
I6	6	4	7	8	9		8	2
I7	3	6	5	4	6	1		1
I8	6	8	9	9	8	7	8	
Rj	32	35	33	49	39	19	37	8
$(R_j - R_{j_{\text{mean}}})^2$	0,25	12,25	2,25	306,25	56,25	156,25	30,25	552,25

$R_{j_{\text{mean}}}$	=	31,5
Δ_{max}	=	3402
Δ	=	1116
K	=	32,8%